SUPPORTING ADULT GUIDANCE

The constant ratio

Introduction to Trigonometry. For any supporting adult — teacher, practitioner, parent or carer. **You do not need to be a mathematics specialist to support this lesson.**

What this lesson does

The learner discovers the foundational idea of trigonometry: in a right-angled triangle, once an angle is fixed, the **ratio** of two sides is **constant** whatever the size. The emphasis is on **similarity** (equal corresponding angles and corresponding sides in the same ratio), on the correct use of **scale factor**, and on seeing the ratio as a property of the **angle**.

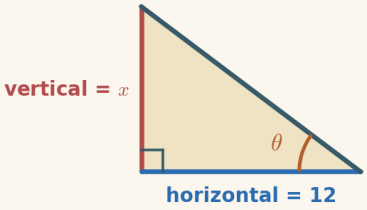
Driving Question: *Why does the ratio of two sides stay the same when only the size of a right-angled triangle changes?*

The mathematics, in plain terms

This lesson lays the single foundation on which sine, cosine and tangent are later built. When a right-angled triangle is **enlarged** — every side multiplied by the same **scale factor** — the side lengths change together but the angle does not, and neither does the ratio of any two sides. So for a fixed angle, **vertical ÷ horizontal is a constant**: a 3-4-5 triangle gives 3 ÷ 4 = 0.75, and so does a 6-8-10, a 9-12-15, and every enlargement of it. Two right-angled triangles are **mathematically similar** when their **corresponding angles are equal** and their **corresponding sides are in the same ratio** — one is an exact scale-factor enlargement of the other — and equal ratios are exactly what tells you the triangles are similar. The key realisation is that this fixed ratio **belongs to the angle, not to the size**: change the angle and the ratio changes; change only the size and it does not. That is precisely why, once the ratios for each angle are known, a single measured side is enough to find the others — the learner gets a first taste of this by using a known ratio to find a side that was never measured. The words matter here: we say **scale factor** (not 'size'), because it is the side **lengths** that are multiplied; we use **vertical** and **horizontal** for the two shorter sides (they get their formal names — opposite, adjacent — in the next lesson); and we use **similar** in its exact mathematical sense. Meeting this language correctly now sets up the whole unit.

The worked method the learner sees

These are the two worked examples the learner meets on screen, with the lesson's exact method and notation. They are here for consistency, not because the mathematics is demanding — whether you teach mathematics or are meeting it afresh alongside the learner, working from the same steps and the same language keeps your support aligned with what is on the screen, and lets you point back to a line the learner has already seen. Skip them freely if the method is already familiar.

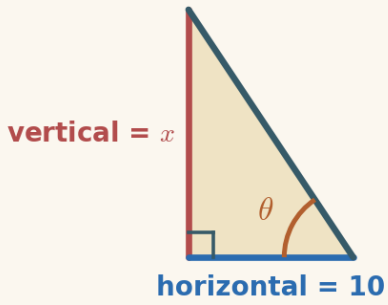


Example 1 — finding a hidden length

Two triangles are the same shape (similar), so they share the same ratio. The first has vertical 3 and horizontal 4. The second has horizontal 12 — find its vertical.

1. the shape's ratio is vertical ÷ horizontal	$\frac{3}{4} = 0.75$
2. same shape, so its ratio is 0.75 too	$\frac{x}{12} = 0.75$
3. rearrange (x is on top, so multiply)	$x = 12 \times 0.75$
4. work it out	$x = 9$

$x = 9$



Example 2 — a steeper shape

A steeper shape has the ratio vertical ÷ horizontal = 1.5 (for example, vertical 6 and horizontal 4). Another triangle of the same shape has horizontal 10 — find its vertical.

1. the shape's ratio is vertical ÷ horizontal	$\frac{6}{4} = 1.5$
2. same shape, so its ratio is 1.5 too	$\frac{x}{10} = 1.5$
3. rearrange (x is on top, so multiply)	$x = 10 \times 1.5$
4. work it out	$x = 15$

$x = 15$

The ratio belongs to the angle

The heart of this lesson is one perception: enlarge a right-angled triangle and the ratio of two sides does not move. It is worth letting that land slowly, because it is genuinely surprising the first time — everything about the triangle changes except this one number. Two threads support it. First, the language of **similarity** and **scale factor**, used precisely: enlarging multiplies the side lengths by a scale factor, similar triangles have equal corresponding angles and sides in the same ratio, and equal ratios are the test for similarity. Second, the shift from 'the ratio is constant' to 'the ratio belongs to the

angle' — change the angle and the ratio changes; change only the scale factor and it does not. Grounding this in the learner's own drawing (draw one triangle, enlarge it, compute the ratio twice) makes the invariance convincing rather than asserted. The lesson ends by using a known ratio to find an unmeasured side, which quietly answers the question 'what is this for?' and points straight at the rest of the unit.

The shape of the lesson

Timings are invitations, not deadlines. A learner may need much longer on one phase and skim another — that is expected. Each Cornerstone is given an honest mathematical role here:

Cornerstone	Its role here	Invitational timing
Connection	Ratio Locks: enlarge a right-angled triangle with the scale-factor slider and watch vertical ÷ horizontal stay constant.	~12 min
Movement	Stack the similar triangles: build a family of similar triangles and check every one shares the ratio.	~12 min
Reflection	Similar or not?: compare two triangles' ratios to decide whether they are similar.	~10 min
Creativity	Find the hidden length: use a known ratio and one side to find a side you cannot measure.	~6 min
Rest	A pause after seeing everything change while one number stays perfectly still.	~3 min
Nutrition	Mode B (intellectual): what the learning feeds — the habit of looking for what stays the same amid change.	~2 min

Common misconceptions, and gentle repairs

These are the sticking points to expect. In every case, the aim is a question that returns the thinking to the learner — never to supply the answer.

Saying the triangle 'gets bigger' without meaning the side lengths

Repair: “What exactly are we multiplying by the scale factor — the side lengths, or the area? Here it is every side length.”

Thinking a bigger triangle must have a bigger ratio

Repair: “Work out vertical ÷ horizontal for the small one and the large one. What do you notice?”

Believing two triangles are similar because they are both right-angled

Repair: “Similar needs equal corresponding angles AND sides in the same ratio. Do the ratios match?”

Treating 'similar' as meaning 'roughly alike'

Repair: “In mathematics, similar is exact: corresponding angles equal, corresponding sides in the same ratio.”

Thinking the ratio depends on how big the triangle is drawn

Repair: “Does the ratio change when you only change the scale factor? It depends on the angle, not the size.”

Protecting the support pathway

The lesson offers support in a deliberate order: **Socratic prompt** → **first try** → **hint** → **second try** → **worked solution**. The worked solution for each question stays locked until the learner has used the prompt, used the hint, and checked two answers. This is by design: the steps *are* the learning.

Please hold this line

Do not read the worked solution aloud, or talk a learner straight to the answer, while it is still locked. If they are stuck, use the Reconnection Routes, the prompt and the hint, or ask one of the repair questions above. Arriving at a correct line of reasoning — even slowly — is worth far more than being handed the result.

Reconnection Routes

If an earlier idea is blocking progress, the lesson’s Reconnection Routes offer short recaps of exactly the prerequisites this lesson needs: right-angled triangles and the hypotenuse (from the Pythagoras unit), a ratio written as a division (a ÷ b), and the idea of similar shapes (an enlargement by a scale factor). A learner can choose one independently, or you can invite one without requiring it. Using a route is part of learning, not evidence of falling behind, and it always returns the learner to their place.

Supporting through questions: an example

Keep the language and the invariance with the learner:

- “What is vertical ÷ horizontal for this triangle? And for the enlarged one?”
- “We multiplied every side length by the scale factor — did the ratio change?”
- “Are these two triangles similar? Check: equal ratios, so equal angles.”
- “You know the ratio and one side — can you find the side you cannot measure?”

If they stall, that is the moment for a Reconnection Route or the lesson’s prompt and hint — not for the answer.

Questions you might have

“Why do we say 'scale factor' rather than 'size'?”

Because 'size' is vague — it could mean the side lengths or the area. Scale factor names exactly what happens: every side length is multiplied by the same number. Precise words prevent confusion later.

“What does 'similar' mean here — just 'looks alike'?”

No — it is exact. Two shapes are mathematically similar when their corresponding angles are equal and their corresponding sides are in the same ratio, so one is a scale-factor enlargement of the other.

“How does this connect to the rest of trigonometry?”

Every angle has its own fixed ratios. Once those are known, a single measured side is enough to find the others — which is exactly what sine, cosine and tangent will let the learner do in the coming lessons.

Keeping things regulated and calm

- Keep to one clear step at a time; there is no time pressure and no benefit to speed.
- Treat a wrong or unfinished answer as information for the next step, never as failure.
- Let the learner choose how to record — typed, written, drawn, spoken or annotated.
- If frustration rises, it is fine to pause and return later; the lesson holds their place.
- Avoid any language about age, school year or pace as a judgement of ability.

Recognising progress

Progress is described as Approaching Proficiency → Developing Proficiency → Proficiency → Refining Proficiency across understanding, application, communication and reflection. These describe where a learner is right now, not labels of what they are. Real understanding shown in part is still real understanding, even while other pieces are still settling.

Where they are	What it can look like
Approaching Proficiency	Beginning to engage; names some features and makes a start with support.

Developing Proficiency	Carries out the method with prompts; can explain part of the reasoning.
Proficiency	Works through independently and explains why each step is true.
Refining Proficiency	Applies the idea confidently to a new or unfamiliar case, and could teach it to someone else.

A note on evidence

Evidence can be captured in whatever form suits the learner — a photo of working, a short spoken explanation, an annotated Scratchpad image, or a written solution. There is no single required format. Incomplete work is information for the next connection, never a failure: it tells you exactly where to begin next time. The aim is a true record of the learner's thinking, not a tidy performance.

If a learner is stuck or distressed

Slow the pace and reduce the load: return to one concrete, familiar case and a single small question. Offer a Reconnection Route. Remind them they can pause and come back. A short, successful, low-stakes step rebuilds confidence more reliably than pushing through. Connection before curriculum — always.